Solution exercice 6_1

Bilan hydrique: dV/dt = I - O (I : débit entrant, O : débit sortant)

Or : Variation du stock d'eau négligeable ➔ dV/dt = 0

I = P (P : précipitations)

O = ET + Q ET : évapotranpiration; Q : débit de la rivière)

➔ 0 = P – ET – Q soit : ET = P - Q

Application :

S = 26000 × 10^6 m^2, since 1 km^2 = 10^6 m^2
P = 594 mm/an/m^2 catchment
Q = 170 m^3/s = 5.36 × 10^9 m^3/an = 5.36 × 10^9 / 26 000 × 10^6 = 0.206 m outflow/m^2 catchment = 206 mm outflow/m^2 catchment

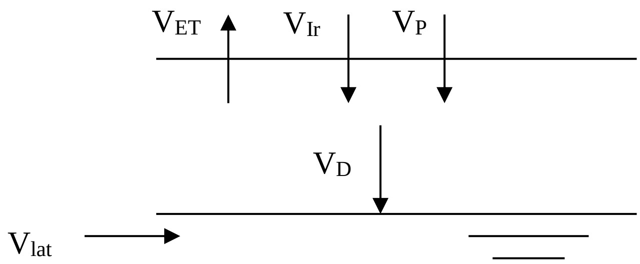
➔ ET = P – Q = 594 – 206 = 388 mm/an per m^2 catchment

Here, precipitation etc. is volume per unit area of land. The answer is per m^2 catchment to emphasize that we are considering per unit area.

Solution exercice 6_2

Bilan hydrique : $\Delta S = P - R + Ir + G - D - ET \quad \Rightarrow ET = P - R + Ir + G - D - \Delta S$
Variation du stock : ΔS = Stock final – Stock initial

ΔS1 = 241 – 512 = -271 mm ET1 = 336 mm
ΔS2 = 555 – 512 = 43 mm ET2 = 349 mm
ET1 = 66 – 30 + 50 – 21 + 271 = 336 mm
ET2 = 66 – 0 + 390 + 0 – 64 – 43 = 349 mm



Solution exercice 6_3

Il s'agit d'établir l'équation du bilan hydrique entre la surface du sol et la nappe :

Apports : $V_{Ir} + V_P$ Pertes : $V_{ET} + V_D$ (Ir: irrigation; P: pluie; D: percolation)

soit : $V_{Ir} + V_P - V_{ET} - V_D = 0$ (puisque le stock hydrique dans la zone non saturée ne varie pas)

Easiest to consider two zones, unsaturated and saturated, and to consider the transfer between them.

V_{Ir} = 2100 m^3/an
V_P = 1200 mm/an = 1200 × 10^{-3} m/an × 100^2 m^2 = 12000 m^3/an
V_{ET} = 1350 mm/an = 1350 × 10^{-3} m/an × 100^2 m^2 = 13500 m^3/an
2100 m^3/an + 12000 m^3/an – 13500 m^3/an - V_D = 0
V_D = 600 m^3/an

Volume d'eau apporté latéralement: V_{lat} = 2100 – 600 = 1500 m^3/an.

Solution exercice 6_4

This exercise is best done in Excel, as in the accompanying file.

Solution exercice 6_5

Kostiakov: $I(t) = a\, t^b$ et: $i(t) = a\, b\, t^{b-1}$

a) $I(15) = 1.3$ cm $= a\, 15^b$
$I(160) = 7$ cm $= a\, 160^b$

$\rightarrow$ $a = 0.19$ et $b = 0.71$

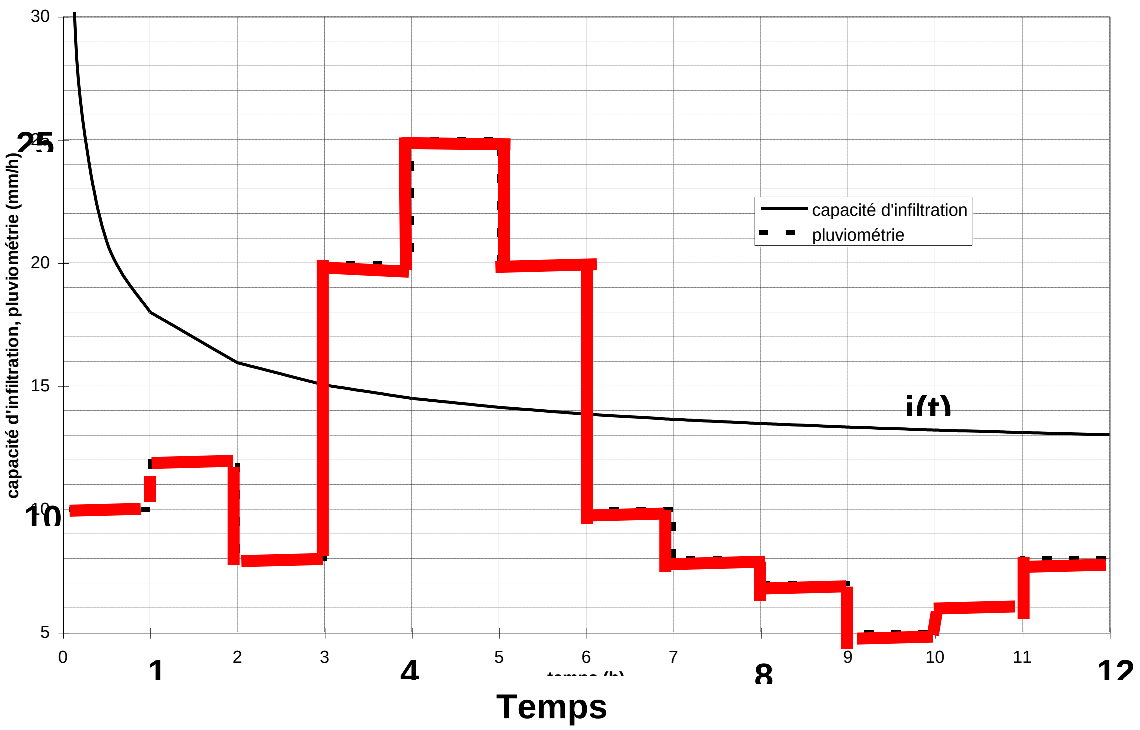
b) $I(300) = 0.19 \times 300^{0.71} = 10.9$ cm

$i(300) = 0.0258$ cm/min

Solution exercice 6_6

a)

t (h)	1	2	3	4	5	6	7	8	9	10	11	12
i (mm/h)	18	16	15	14.5	14.1	13.8	13.6	8	13.3			13



b) ruissellement pour $i_c < P$ ➔ le ruissellement débute au bout de 3 heures

c) lame ruisselée = 22 mm = 220 m^3/ha ➔ 1’650’000 m^3

d) $$C = \frac{\text{lame ruisselée}}{\text{pluie brute}} = \frac{22}{139} = 0.16$$

Solution exercice 6_7

- Profondeur z_f du front d'humidité après 1h d'infiltration : $t = \frac{\theta_s - \theta_i}{K_s}\left[z_f + h_f \ln\left(1 - \frac{z_f}{h_f}\right)\right]$

$1 = \frac{0.4-0.1}{8.32}\left[z_f - 11\ln\left(1+\frac{z_f}{11}\right)\right]$ Par itération, on trouve : z_f = environ 45 cm

- Vitesse v_f de progression du front d'humidité :

$v_f = \frac{dz_f}{dt}$ Or : $I = (\theta_s - \theta_i) z_f$ ➔ $z_f = \frac{I}{(\theta_s - \theta_i)}$

soit : $v_f = \frac{dz_f}{dt} = \frac{1}{(\theta_s - \theta_i)}\frac{dI}{dt} = \frac{1}{(\theta_s - \theta_i)} i$ Or : $i = K_s\left(1 - \frac{h_f}{z_f}\right) = 8.32\left(1 + \frac{11}{45}\right) = 10.35\,\text{cm/h}$

➔ $v_f = \frac{1}{(0.4-0.1)} 10.35 = 34.5\,\text{cm/h}$

- Infiltration cumulée I : $I = (\theta_s - \theta_i) z_f = 13.5\,\text{cm}$

d) The Green-Ampt sorptivity is given by : $S_{GA} = (2\Delta h \Delta\theta K_s)^{1/2}$,

with $\Delta h = h_{surface} - h_{front} = 0 - 11 = 11$

$\Delta\theta = \theta_s - \theta_i = 0.4 - 0.1 = 0.3$

So: $S_{GA} = (2 \times 11 \times 0.3 \times 8.32)^{1/2} = 7.41$ cm/hr$^{1/2}$

Solution exercice 6_8

a) The flux of water into the soil is given by Darcy's law(z axis pointing downwards):

$$q = -K_s \frac{dH}{dz} = -K_s \frac{d(h-z)}{dz} = -K_s \left(\frac{h_{front} - h_{surface}}{z_f - 0} - 1\right) = -K_s\left(\frac{h_f - h_s}{z_f} - 1\right)$$

Also, $I(t) = z_f(t)\Delta\theta$. The head on the surface, h_s, is a function of time, t, i.e.,

$$h_s(t) = h_s(0) - I(t).$$

Recall that $\Delta\theta = \theta_s - \theta_i$. Darcy's law then becomes:

$$q = K_s\left[1 + \frac{\Delta h - I(t)}{I(t)/\Delta\theta}\right],$$

where $\Delta h = h_s(0) - h_f$. We also have:

$$q = \frac{dI}{dt}.$$

These expressions are equated to yield the governing equation for I:

$$\frac{1}{K_s}\frac{dI}{dt} = 1 - \Delta\theta + \frac{\Delta\theta\Delta h}{I}.$$

The solution to this differential equation satisfying I(0) = 0 is:

$$I(t) = K_s t(1-\Delta\theta) + \frac{\Delta\theta\Delta h}{1-\Delta\theta}\ln\left[1+\frac{I(t)(1-\Delta\theta)}{\Delta\theta\Delta h}\right].$$

In terms of z_f, this equation is:

$$z_f(t)\Delta\theta = K_s t(1-\Delta\theta) + \frac{\Delta\theta\Delta h}{1-\Delta\theta}\ln\left[1+\frac{z_f(t)(1-\Delta\theta)}{\Delta h}\right].$$

b) Expand the ln term (two terms are needed, higher order terms disappear in the limit $z_f \rightarrow 0$, which is the expansion sought):

$$z_f(t)\Delta\theta = K_s t(1-\Delta\theta) + \frac{\Delta\theta\Delta h}{1-\Delta\theta}\left\{\frac{z_f(t)(1-\Delta\theta)}{\Delta h} - \frac{1}{2}\left[\frac{z_f(t)(1-\Delta\theta)}{\Delta h}\right]^2\right\}.$$

So:

$$z_f(t)\Delta\theta = K_s t(1-\Delta\theta) + \Delta\theta z_f(t) - \frac{1}{2}\Delta\theta z_f(t)\frac{z_f(t)(1-\Delta\theta)}{\Delta h},$$

thus:

$$0 = K_s t(1-\Delta\theta) - \frac{1}{2}\Delta\theta z_f(t)\frac{z_f(t)(1-\Delta\theta)}{\Delta h}.$$

Let: $I(t) = \Delta\theta z_f(t)$, then

$$0 = K_s t(1-\Delta\theta) - I(t)^2\frac{1-\Delta\theta}{2\Delta\theta\Delta h},$$

or

$$I(t) = \sqrt{2K_s\Delta\theta\Delta h t} = S_{GA}\sqrt{t},$$

(since $I = St^{1/2}$ as $t \rightarrow 0$) and so

$$S_{GA} = \sqrt{2K_s\Delta\theta\Delta h},$$

which is the Green-Ampt sorptivity.

c) Use S_{GA} to replace Δh in:

$$I(t) = K_s t(1-\Delta\theta) + \frac{\Delta\theta\Delta h}{1-\Delta\theta}\ln\left[1+\frac{I(t)(1-\Delta\theta)}{\Delta\theta\Delta h}\right].$$

This gives:

$$I(t) = K_s t(1-\Delta\theta) + \frac{S^2}{2K_s(1-\Delta\theta)}\ln\left\{1+\frac{I(t)}{S^2/[2K_s(1-\Delta\theta)]}\right\}.$$

d) The case of a fixed head on the surface leads to the formula:

$$\mathrm{I(t)} = \mathrm{K_s t} + \frac{\mathrm{S^2}}{\mathrm{2K_s}} \ln \left\{ 1 + \frac{\mathrm{I(t)}}{\mathrm{S^2/[2K_s]}} \right\}.$$

The falling head on the surface acts to reduce the effect of gravity by the factor 1 - $\Delta\theta$, i.e., instead of K_s, the falling head case has $K_s(1 - \Delta\theta)$.

e) The initial height of water on the surface is $h_s(0)$. The time for this amount of water to enter is given by $I(t) = h_s(0)$, i.e., from the Green-Ampt formula above:

$$\mathrm{t} = \frac{1}{\mathrm{K_s}(1-\Delta\theta)} \left\{ \mathrm{h_s}(0) - \frac{\mathrm{S^2}}{2\mathrm{K_s}(1-\Delta\theta)} \ln \left\{ 1 + \frac{\mathrm{h_s}(0)}{\mathrm{S^2}/[2\mathrm{K_s}(1-\Delta\theta)]} \right\} \right\}.$$

This is the maximum time for which the Green-Ampt formula is valid for the falling head case, since all the water on the surface has drained into the soil.

Solution exercice 6_9

(a) The soil is initially dry, and then is placed in the water. The soil in that is below the soil surface is fully saturated. Due to capillary rise, the zone in the soil column above the water surface is nearly saturated. The height of this nearly saturated zone is, approximately, S^2/K_s, since S^2 has dimensions L^2/T and K_s has dimensions L/T. The saturated hydraulic conductivity is used since the zone above the watertable is nearly saturated. Note that there is no other characteristic length that can be applicable given that there are only two parameters involved. Similarly, the time scale for the water to move into the soil is, again from dimensional analysis, S^2/K_s^2.

(b) The same reasoning as in part (a) applies, except that the hydraulic conductivity is now given by $K(\theta_0)$. The physical interpretation is different, however. So long as the vertical dimension is appreciably less than $S^2/K(\theta_0)$., the variations in the vertical due to gravity are dominated by capillarity.

In general, the scalings given above reflect the two forces that control water movement in soil, capillarity and gravity, which are quantified by S and K, respectively.